Synopsis

Title of the Research: Mathematical studies of discrete-time ecological systems with Allee effect

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Date of Registration: 28.11.16

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Keywords: Discrete-time predator-prey model; Allee effect; Bifurcation; Chaos; Stability.

Resesarch Proposal

Over the last few years, study of population dynamics through mathematical modeling becomes more interesting to the theoretical ecologists and mathematical biologists due to it's rich dynamics and wide applications [1; 2]. For modeling the population dynamics, two types of mathematical models are popular, namely, continuous-time models and discrete-time models. Continuous-time models are described by differential equations and discrete-time models are described by difference equations. In a small population size or the population having no overlapping generation, the discrete-time models are more suitable than continuous ones; for example, many species of insect have no overlap in their successive generations, and hence their populations obey discrete-time behavior. Also, discrete-time system gives richer dynamics compare to the continuous-time system. For instance, in a single-species discrete-time system can exhibit chaos [1] and more complex dynamical behavior whereas in a continuous-time system minimum three species are needed for exhibiting chaos [2].

Allee effect is an important phenomenon in ecological systems, both in discrete-time modeling [3] and continuous-time modeling [4]. The concept of Allee effects was first explained by W. C. Allee [5], where the component Allee refers to the positive correlation between species individual fitness and population size/density, and the demographic Allee effect is the positive correlation between population size/density with per-capita growth rate (pgr) [6]. There are several mechanisms such as complication in finding mates, reproductive facilitation, predation, environment conditioning, inbreeding depressions, etc. behind the Allee effect. Allee effects mainly classified into two ways: strong and weak Allee effects [6]. There is a threshold population level for the strong Allee effect such that the species become extinct below this threshold population density. On the other hand, the weak Allee effect occurs when the growth rate reduces but remains positive at low population density. Empirical evidence of Allee effects has been observed in many natural species, for example, plants [7], insects [8], marine invertebrates [9], birds and mammals [10], etc. Allee effect has received substantial attention from both theoretical and applied ecologists, especially conservation biologists [11].

The stability analysis of the predator-prey system has great importance in understanding its biological relevance and may be helpful in conservation management. Allee effect is an ecological phenomenon, which plays an important role in conservation management. Allee effect could have stabilizing or destabilizing effect on population dynamics.

So it will be interesting to study how dynamical behaviours of a different discrete systems change by the positive density dependence growth factor, i.e., by Allee effects. The aim of the research work is to develop some mathematical model with both strong and weak Allee effect and investigate the models rigorously by using both mathematical and numerical techniques.

We hope successful completion in the research work may generate a considerable interest in theoretical ecology among the researchers.

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